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Calculation of the diffraction effects in architectural acoustics

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ABSTRACT

Realistic modeling of sound propagation in rooms and halls is an important topic of research since 60-s. Unfortunately, current software products based on geometrical ideas do not simulate reverberant sound accurately. They model only direct transmission and specular reflections, while diffraction is either ignored or modeled through statistical approximation. Standard rectangular rooms may possess "billiard" modes, the attenuation mechanism of which is related to the Fresnel diffraction. There is no convenient analytical expression for amplitude of multiply diffracted ray. Existent methods to compute diffraction effects calculate only the first and, sometimes, the second order of diffraction. In this work two-dimensional model problem is investigated to modify the ray tracing method to describe correctly multiple Fresnel diffraction on the edges. The calculation of diffraction is made by solving numerically the parabolic equation of theory of diffraction for each ray. This method has reasonable accuracy and efficiency. The results obtained by the new method are compared with results of FEM, Uniform Theory of Diffraction and Ray Tracing without diffraction.

1. INTRODUCTION

Realistic modeling of propagation of sound in virtual environments is important for applications. Current interactive sound propagation methods are based on Geometrical Acoustic (GA) techniques [1, 2]. Unfortunately, current software products based on geometrical ideas do not simulate reverberant sound accurately. Standard rectangular rooms may possess "billiard" modes [3], the attenuation mechanism of which is related to the Fresnel diffraction. There is no convenient analytical expression for amplitude of multiply diffracted ray. Existent methods to compute diffraction effects calculate only the first and, sometimes, the second order of diffraction. In this work two-dimensional model problem is investigated to modify the ray tracing method in order to describe correctly multiple Fresnel diffraction on the edges.

2. FORMULATION OF PROBLEM

In the present study we suppose that the walls are ideally flat and hard; the room geometry is chosen as rectangu1ar. In an exact formulation of the problem, we suppose that the wa1ls are rigid borders of a certain rectangular area in an infinite space. The windows are obtained by removing certain areas of the boundary. An acoustic wave is diffracted on the windows and its energy is carried away into open space. Such an exact formulation will be replaced below by a simplified one, which, however, does not qualitatively change the results. We consider a two-dimensional model problem. Windows can be apertures in the wa1ls, as well as perfectly matching absorbing elements [4]. For the given family of rays we apply the reflection method. We reflect the room relative to its walls and do the same with the obtained reflections, etc. We suppose that the walls are ideally hard. As well, as it hits a window (or their reflections), the ray is fully absorbed. In this way, there are no walls in the constructed system of reflections and the windows are ideally absorbing screens. As a result of using the reflection method, a virtual room consisting of two walls becomes a waveguide like an aperture line. So, we have a problem of diffraction on a system of inclined screens.

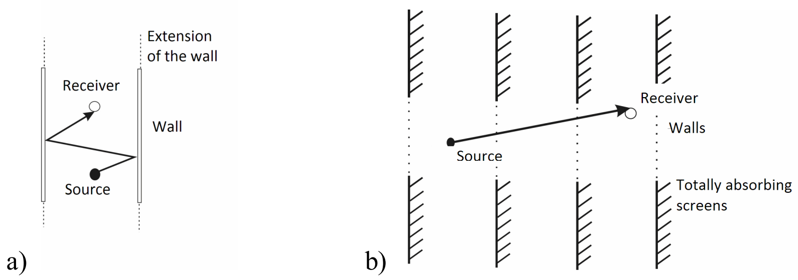


Figure 1. a) Reflection of the ray at the walls and b) method of reflections

3. ALGORITHM

3.1. Selection of the edges affects acoustic field

Let us consider auxiliary problem: a ray travels from source to receiver through an aperture line, which is an array of totally absorptive screens. Selection is a separation of all edges into "close" and "remote" with respect to any given ray. "Close" edges affect the ray, i.e. it is necessary to take them into account when describing the diffraction process accompanying propagation of this ray. "Remote" edges are far enough from the ray, and they do not affect the propagation process. We compute the path of a direct ray from the source to the receiver, and then for each edge we compute the diffracted path, i.e. the length of the path source-edge-receiver. If the diffracted path is more than $\lambda/2$ bigger than the direct path, we mark the edge as "remote". Otherwise we mark the edge as "close". So, we have an array of “close” edges that can be represented as an ellipse with its focuses in the points of source and receiver. The dimension of the minor axis of an ellipse is about the dimension of Fresnel zone $a \sim \sqrt{\lambda L}$, where λ is a wavelength, L is a source-receiver distance (fig.1). If the edge of screen hits into the ellipse this edge should be taken into account. If the edge of screen doesn’t hit the ellipse, such edges are rejected and the diffraction effects are not calculated for them.

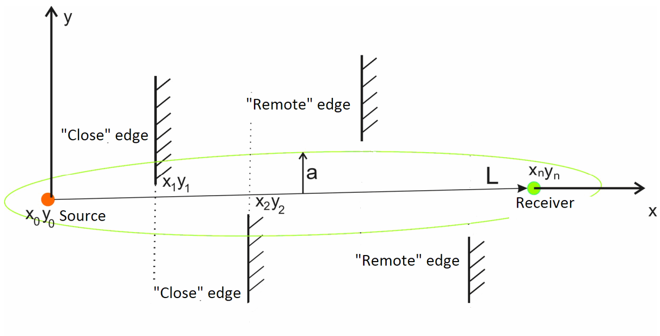


Figure 2. Close and remote edges

3.2. Parabolic equation

In the free space, the Helmholtz equation is fulfilled:

$$\Delta u + k_0^2 u = 0 \tag{1}$$

Where u – potential, $k_0 = \frac{\omega}{c_0}$ - wave number, ω -circular frequency, c_0 - sound velocity. We use parabolic approach that allows simplifying the boundary conditions in comparison with Helmholtz equation. The contribution of rays, scattered at the big angles isn’t calculated. Their contribution is small to negligible and does not influence the accuracy of the result. The given simplification, of course, strongly influences the scattering of rays at large ang1es (it influences Keller diffraction), but has a1most no effect on Fresnel diffraction. In this approach the acoustic potential will obey the equation Eq. (2).

$$\frac{\partial^2 u}{\partial y^2} + 2ik_0 \frac{\partial u}{\partial x} = 0 \tag{2}$$

The boundary conditions are

$$u(x = x_{cm}n + 0, y_{cm}) = 0; \quad u(0, y) = \delta(y); \quad u(x < 0, y) = 0 \tag{3}$$

Using the Green’s function of the parabolic problem for the entire plane, namely $\hat{G}(x, y) = \sqrt{\frac{k_0}{2\pi x}} e^{\frac{ik_0 y^2}{2x} - \frac{i\pi}{4}}$, we can find the potential u at any point of the plane with y > 0:

$$u(x_1, y_1) = \int_{\Gamma_0} u(x_0, y_0)\hat{G}(x_1 - x_0, y_1 - y_0)dy_0 \tag{4}$$

$$u(x_2, y_2) = \iint_{\Gamma_0 \Gamma_1} u(x_0, y_0)\hat{G}(x_1 - x_0, y_1 - y_0)G(x_2 - x_1, y_2 - y_1)dy_1 dy_0 \ldots \tag{5}$$

Γ_0, Γ_1 – surfaces of integrating, those are extension of the edges, which lies in the two-dimensional Frenels zone.

3.3. Structure of the Ray Tracing program with parabolic propagation kernel

The Ray Tracing program with calculation of diffraction by solving the parabolic equation consists of the following steps. On the first step the geometry of the two-dimensional room, the original coordinates of the point of source and the direction of the first ray was set. After the ray was traced the information about the walls hit by the ray was collected. This information includes the distance from the edge of the wall to the ray, the length of the ray, the location of the edge relative to the ray, and the quantity of the hits with the walls of one ray. The received information was proceeded to the next step – the numerical solution of the parabolic equation and calculation the pressure on the receiver.

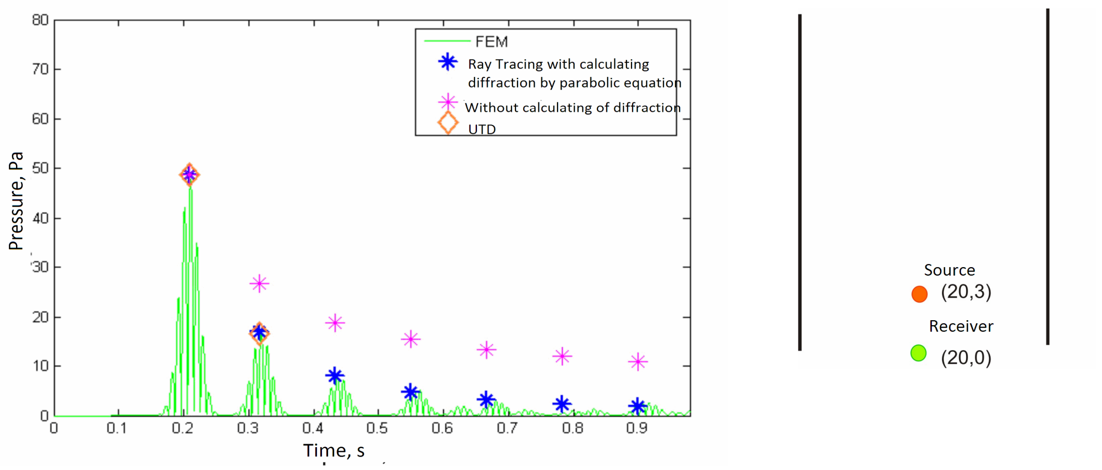
$$p = \rho \frac{du}{dt} = -i\omega\rho u \tag{6}$$

where ρ – density of air, p – pressure, ω – circular frequency.

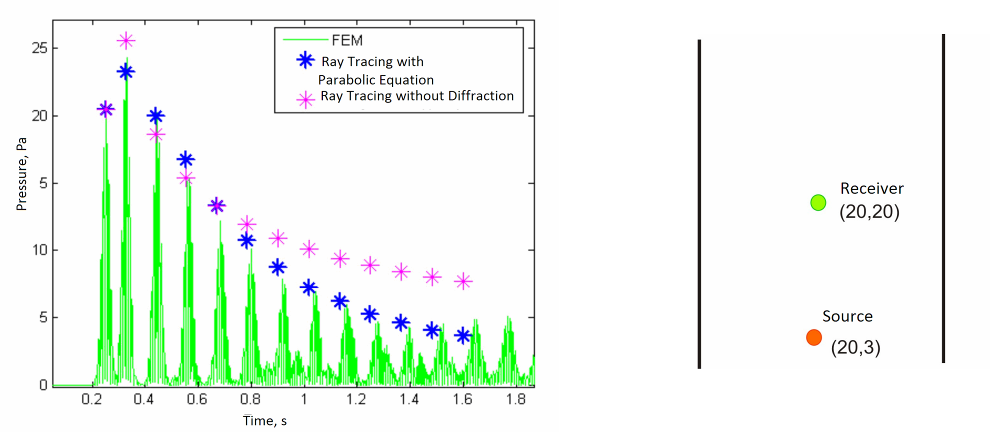
4. NUMERICAL RESULTS

One of the most accurate methods of calculating acoustic field is Finite Element Method [5]. We developed a program that realized this method. In addition to FEM, we use Ray Tracing without diffraction [6] and Ray Tracing with UTD [7] for comparison.

Investigation is carried out for two-dimensional problem. There are two parallel walls. The length of these walls is 60 meters, distance between walls is 40 meters, and the frequency is 50 Hz. These parameters are selected to compare with FEM. Source is in the middle between the walls, the distance between source and edge is 3 m, so the coordinates of the source are (20, 3). Some locations of receiver are considered: a) receiver is at the level of walls (coordinates (20,0)); b) receiver is inside the walls (coordinates (20,20)) (Figure 3). From a) we can see, that UTD can give results only for the first and the second orders of diffraction, Ray Tracing without diffraction gives incorrect result, but FEM and Ray Tracing with Parabolic equation give the same results. From b) we can see that Ray Tracing without diffraction, FEM and Ray Tracing with Parabolic equation give the same results when the order if scattering is small. But in greater orders of scattering FEM and Ray Tracing with Parabolic equation give the same results, different from results of Ray Tracing without diffraction.



a)



b)

Figure 3. Comparison of time dependences, receives by Ray Tracing with consideration of diffraction by solving parabolic equation, Ray Tracing without diffraction, UTD and FEM.

5. CONCLUSIONS

1. We presented a prototype of Ray Tracing program which calculates amplitude of every ray with accurate consideration of diffraction effects.
2. We reformulated the problem for each ray from propagation with reflections to propagation in aperture line. It simplifies the selection of the edges affecting acoustic field and allows using parabolic approach.
3. We selected the edges to simplify the solving of the diffraction problem in a virtual two-dimensional room by using of Ray Tracing.
4. We used parabolic equation to simplify the solving of the problem of Fresnel diffraction. So, this program based on GA allows to calculate diffraction of the first, second and higher orders. Calculating of high orders of diffraction distinguish our program from existing programs that use methods of geometric acoustics and can't calculate high orders of diffraction. Utilizing the parabolic equation gives a good agreement with Finite Element Method and it shows high accuracy of developed method. The method under consideration involves less time and budgets then FEM and it is more accurate then Ray Tracing without calculating of diffraction effects.

6. REFERENCES

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