Sound decay in a rectangular room with specular and diffuse reflecting surfaces

Nikolay Kanev
Andreyev Acoustics Institute, Moscow, Russia.

Summary
Sound decay in rooms is usually defined by absorption and scattering properties of surfaces. In some cases sound decay calculation is a rather simple task. But in most problems we deal with nonuniformly absorption distribution on the surfaces, nondiffuse reflections, nonergodic rooms and other obstacles preventing the exact analytical solution. Simulation procedures can help us in practical cases when we investigate a certain room. There are some different models for calculation of diffuse sound field, whereas specular reflections can be taken into account in a simple way. In the present paper we propose analytical model for sound decay calculation in a rectangular room. In this model sound field in the room is separated into two components. First component describing specular reflected sound field is given by a sum of specular reflections. Second component describing diffusely reflected sound field is a solution of a differential equation. Without diffuse reflections sound decay in the room with nonuniformly absorption distribution is always nonexponential. If the diffuse reflection coefficient is sufficient then sound decay is exponential and close to Sabine's law. Separation of sound field into secular reflected and diffusely reflected components permits to define what of them is dominant. Conditions under which sound decay is mainly described by diffuse reflections can be estimated from the model as well.

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1. Introduction

The reverberation time is generally recognized as the most important acoustical attribute for any kind of rooms. It characterizes a sound decay rate on condition that a decay curve is assumed to be close to exponent. Many formulas have been proposed for predicting the reverberation times [1-3].

A necessary condition of existence for the reverberation time is that the sound field in the enclosure is diffuse and sound decay is close to exponent. For that the enclosure must be sufficiently randomizing [4]. Randomization of the sound field can be provided by the enclosure shape or by the roughness of its surfaces. In the enclosure with these properties any initial sound field becomes diffuse or homogeneous with time. But this condition is not sufficient for the enclosures with a high nonuniform distribution of absorption. Significant deviations from a pure exponential decay law take place in strongly chaotic rooms with absorption localized in certain region of the rooms [5].

An extreme case is a nonrandomizing enclosure with nonuniform distribution of absorption without any scattering obstacles and surfaces. Both properties can result in great deviation of sound decay from the exponential law. For example [6] sound energy in a rectangular room with an absorbing ceiling is proportional to $1/t$. Taking into account scattering properties of enclosure walls permits to achieve more randomized sound field. In order to describe scattering by the surfaces a diffuse reflection coefficient is usually introduced.

It is convenient to separate sound energy into two components [4,6]. First of them is defined by specular reflected sound energy. Second one is defined by scattered energy. This approach is developed for case when both components are defined by diffuse fields and decay in accordance with exponential law [7]. But in nonrandomizing enclosures nonexponential decays of specular energy have to be taken into consideration.

In this work we consider a nonrandomizing enclosure by the example of a rectangular parallelepiped with specular and diffuse reflecting surfaces. First of all we find a decay law for energy of specular reflection and then investigate the influence of sound scattering by surfaces on decay of total sound energy.

2. Specular reflecting walls

Let consider a rectangular parallelepiped enclosure with dimensions L, D, H. The beginning of the coordinate system is placed in the enclosure corner and the axes are directed along the enclosure edges as shown in Figure 1a. Six walls of the enclosure are numerated in the following way: a number of the wall lying in the plane $z = H$ is 1; a number of the wall lying in the plane $z = 0$ is 2; numbers of the walls lying in the planes $x = L$, $x = 0$, $y = D$, $y = 0$ are 3, 4, 5, 6 respectively. Absorbing properties of the enclosure walls are characterized by specular reflection coefficients γ_i, where i is the wall number. We assume that the walls are smooth and do not diffuse sound. So the absorption coefficients of the walls are equal to $\alpha_i = 1 - \gamma_i$.
Initial sound field in the enclosure is diffuse. In accordance with Kuttruff [3] it means that at any point in the enclosure sound waves are incident from all directions with equal intensity and random phase. So at the moment $t = 0$ the sound field is a superposition of incoherent plane waves with equal amplitudes uniformly distributed on the directional angles φ and θ. Note that the plane waves can be replaced by sound rays or sound particles [4,5] but the following computations are correct for them as well. Sound energy in the enclosure is defined as sum intensity of all waves (or sound particles) and given by

$$E_s(t) = \frac{1}{4\pi} \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left| A(t,\theta,\varphi) \right|^2 \cos\theta d\theta d\varphi , \qquad (1)$$

where $A(t,\theta,\varphi)$ is the amplitude of the wave propagating in direction determined by the angles φ and θ.
In order to investigate the sound decay process we apply the image room method [3]. The enclosure is continuously mirrored as whole at its walls. Images of the original enclosure fill whole space without leaving uncovered regions and without any overlap. The pattern of the image enclosures in the plane xz is shown in Figure 1b and continues in a similar manner in the y-direction perpendicular to the drawing plane. Initial sound field is mirrored in entire space as well. Finally, we obtain diffuse sound field in all space with the same properties as initial sound field in the original enclosure has. We suppose that an initial condition for the mirrored sound field is

$$A(0,\theta,\varphi) = 1 , \qquad (2)$$

In the proposed model we need no longer consider reflection of sound from the walls. Each reflection is substituted by the intersection of the planes consisting of the wall images by the wave traveling towards the original enclosure. The distance between the neighbour image planes perpendicular to the axis x (y or z) is equal L (D or H). After any intersection the wave loses a fraction of its intensity corresponding to the wall absorption coefficient.
Sound energy in the enclosure at the moment t is

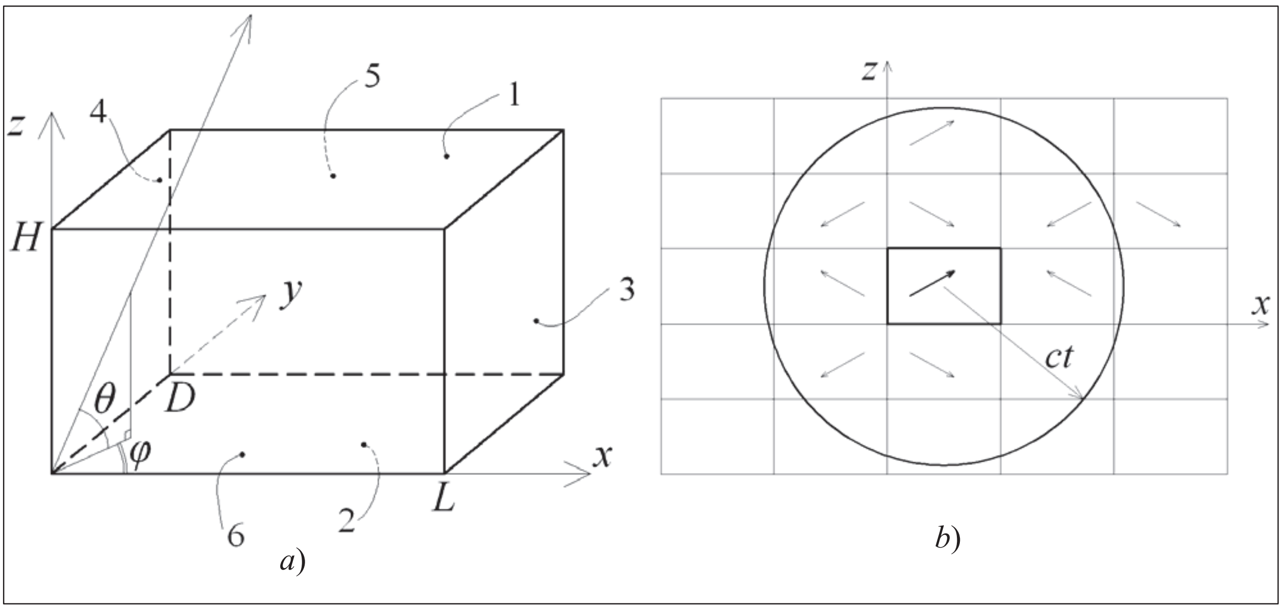

Figure 1. A rectangular enclosure (a) and mirrored enclosures in the plane xz (b).

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defined by the waves passing distance ct towards the enclosure, where c is speed of sound. In order to find intensity reduction of the wave due to walls absorption we have to count the intersections with the image planes. For the sake of simplicity we calculate sound energy at the point $(0,0,0)$, i.e. at the enclosure corner. The wave with the directional angles φ and θ travels distance $|ct\sin\theta|$ along the axis z in time t. Along the axes x and y it travels distances $|ct\cos\theta\cos\varphi|$ and $|ct\cos\theta\sin\varphi|$ respectively. The number of reflections n_i from the wall with number i in time t is equal

$$n_{1,2}(t,\theta,\varphi) = \frac{ct}{2H}|\sin\theta|, \tag{3}$$

$$n_{3,4}(t,\theta,\varphi) = \frac{ct}{2L}|\cos\theta\cos\varphi|, \tag{4}$$

$$n_{5,6}(t,\theta,\varphi) = \frac{ct}{2D}|\cos\theta\sin\varphi|. \tag{5}$$

Intensity reduction of the wave due to absorption on the walls is equal

$$|A(t,\theta,\varphi)|^2 = \prod_{i=1}^{6}\gamma_i^{\,n_i(t,\theta,\varphi)} = \exp\sum_{i=1}^{6} n_i(t,\theta,\varphi)\ln\gamma_i .$$

Total sound energy can be found from equation 1 by integration by the angles φ and θ

$$E_s(t) = \frac{1}{4\pi}\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\sum_{i=1}^{6} n_i(t,\theta,\varphi)\ln\gamma_i}\cos\theta d\theta d\varphi . \tag{6}$$

Equation 6 defines the energy decay law in the enclosure. If all walls are absolutely reflecting and their reflection coefficients are equal to $\gamma_i = 1$ then sound energy in the enclosure remains constant.

Let us consider the enclosure with one absorbing wall. Suppose that $0 < \gamma_1 < 1$ and $\gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = \gamma_6 = 1$. Equation 6 gives

$$E_s(t) \approx -\frac{2H}{ct\ln\gamma_1} \sim \frac{1}{t},\; t >> \frac{H}{c\ln\gamma_1}. \tag{7}$$

Sound decay is inversely proportional to time. Equation 7 coincides with Kuttruff's result [3] for the similar enclosure.

Suppose that two nonparallel walls can absorb sound waves. If the reflection coefficients of the walls are equal $0 < \gamma_1, \gamma_3 < 1$ and $\gamma_2 = \gamma_4 = \gamma_5 = \gamma_6 = 1$ than from equation 6 we can find

$$E_s(t) \approx \frac{8HL}{\pi\ln\gamma_1\ln\gamma_3(ct)^2} \sim \frac{1}{t^2}, \tag{8}$$

for $t >> H/c\ln\gamma_1$ and $t >> L/c\ln\gamma_3$.

Sound decay is inversely proportional to squared time. Sound energy decays faster then in the enclosure with one absorbing wall.

In general case all walls can absorb sound waves. For $0 < \gamma_i < 1$, $i = 1...6$ we can find from Equation 6 for $t >> H/(c\ln\gamma_1\gamma_2)$, $t >> L/(c\ln\gamma_3\gamma_4)$ and $t >> D/(c\ln\gamma_5\gamma_6)$

$$E_s(t) \approx \frac{2}{\pi t^2}\frac{le^{-lt} + de^{-dt} + he^{-ht}}{ldh}, \tag{9}$$

where $d = -\frac{c}{2D}\ln\gamma_5\gamma_6$, $l = -\frac{c}{2L}\ln\gamma_3\gamma_4$,

$h = -\frac{c}{2H}\ln\gamma_1\gamma_2$.

We can see from equation 9 that sound decay is not exponential under any absorption distribution on the walls.

Physical sense of l, d and h is that they characterize exponential decay rates of sound propagating along the axes x, y and z respectively. They depend on both the distance between parallel walls and the absorption coefficient of these walls. The sum in equation 9 can be separated into three summands each of them describes sound energy decay along one of the axis. Sound propagating along the axis with maximal exponential decay rate is absorbed faster then along two other axes. Minimum exponential decay rate determines sound decay at $t \to \infty$. If $l < d, h$ then the slowest sound decay is along the axis x and the sound energy is found from equation 9 at $t \to \infty$

$$E_s(t) \approx \frac{2}{\pi dh}\frac{e^{-lt}}{t^2}. \tag{10}$$

Comparing equations 7, 8 and 10 we can see that the sound decay becomes faster with increasing of number of absorbing walls.

As an example let us consider a rectangular enclosure with maximal dimension H and other dimensions $D = 0.7H$, $L = 0.5H$ and introduce dimensionless time $\tau = ct/2H$ for three different distributions of the sound absorption on its walls. In first case all absorption is concentrated on one wall with the number $i = 1$. The absorption coefficient of this wall is equal $\alpha = 0.7$. In second case the absorption is distributed on two nonparallel walls with the numbers $i = 1$ and $i = 5$.

Their absorption coefficient is $\alpha = 0.29$. In third case the absorption is distributed on three nonparallel walls which absorption coefficients are equal $\alpha = 0.16$. In all cases the mean values $\overline{\alpha}$ of the absorption coefficient are the same and equal $\overline{\alpha} = 0.08$. Classical Sabine's law of sound energy decay is given by

$$E(t) = \exp\left(-\frac{ct\overline{\alpha}S}{4V}\right), \qquad (11)$$

where V is the enclosure volume and S is the area of all walls. Decay curves for there absorption distributions on the walls calculated by equation 6 and Sabine's decay curve calculated by equation 11 are shown in Figure 2. As we can see the decay curves for nonuniform absorption differ strongly from the exponential Sabine's law. Whereas the decay curve for uniform absorption is closest to the exponent but does not coincide with it at $\tau >> 1$.

3. Scattering walls

In order to take into account scattering properties of the wall we introduce a diffuse reflection coefficient δ which is defined as the ratio of scattered sound energy to incident sound energy. It is connected with the absorption coefficient α and the specular reflection coefficient γ by

$$\alpha + \delta + \gamma = 1. \qquad (12)$$

Let us separate sound energy in the enclosure into two components. First component is specular energy $E_s(t)$, which is determined by equation 6. Second component is diffuse energy $E_d(t)$, which is formed by scattered energy. The initial sound field is defined only by specular energy, diffuse energy is equal zero. At every reflection the fraction δ of incident energy transforms into diffuse energy from specular energy. Absorption of diffuse energy is described by ordinary exponential law, which is correct for diffuse sound field. Suppose that diffuse energy decays in accordance with the Sabine's law given by equation 11.

If after any reflection from the wall with the number i the wave with directional angles φ and θ has energy $E_s(t,\theta,\varphi)$ defined by previous specular reflections, then before this reflection specular energy is equal $E_s(t,\theta,\varphi)/\gamma_i$. So scattered energy during this reflection is equal $E_s(t,\theta,\varphi)\delta_i/\gamma_i$. At the time interval from t to $t+dt$ the number of reflection from each wall is equal $\overline{n}_i dt$, where $\overline{n}_i = dn_i/dt$. According to equations 3-5 $\overline{n}_i$ does not depend on time.

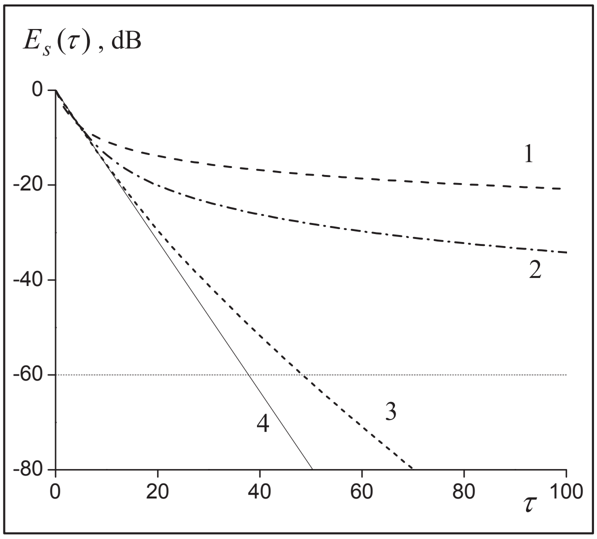

Figure 2. Sound energy decays in the rectangular enclosure with dimensions $H : D : L$=1:0.7:0.5 with one absorbing wall (1), two absorbing walls (2), three absorbing walls (3) in comparison with the Sabine's law (4).

Increase in diffuse energy due to the wave with directional angles φ and θ is defined by all reflections in the time interval $(t, t+dt)$

$$dE_d(t,\theta,\varphi) = \sum_i \overline{n}_i(\theta,\varphi)\frac{\delta_i}{\gamma_i}E_s(t,\theta,\varphi)dt. \qquad (13)$$

Total increase of diffuse energy is given by integration by all angles

$$dE_d^+(t) = \frac{1}{4\pi}\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{t\sum_i \overline{n}_i(\theta,\varphi)\ln\gamma_i} \times$$
$$\times \sum_i \left(\overline{n}_i(\theta,\varphi)\frac{\delta_i}{\gamma_i}\right)\cos\theta d\theta d\varphi dt \qquad (14)$$

In accordance with the Sabine's law diffuse energy decays as $e^{-t/T}$, where $T = 4V/\overline{\alpha}cS$. Reduction of diffuse energy at the time interval dt is equal

$$dE_d^-(t) = -\frac{E_d(t)}{T}dt. \qquad (15)$$

From equations 14 and 15 we can find the differentioal equation for diffuse energy

$$\frac{dE_d}{dt} = -\frac{E_d}{T} + \int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{A(\theta,\varphi)t}B(\theta,\varphi)d\theta d\varphi \qquad (16)$$

with the initial condition

$$E_d(0) = 0, \qquad (17)$$

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where

$$A(\theta,\varphi)=\sum_i \overline{n}_i(\theta,\varphi)\ln\gamma_i\,,$$

$$B(\theta,\varphi)=\frac{1}{4\pi}\sum_i\left(\overline{n}_i(\theta,\varphi)\frac{\delta_i}{\gamma_i}\right)\cos\theta\,.$$

Solution of equations 16 and 17 is given by

$$E_d(t)=e^{-\frac{t}{T}}\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{e^{\left(A(\theta,\varphi)+\frac{1}{T}\right)t}-1}{A(\theta,\varphi)+\frac{1}{T}}B(\theta,\varphi)d\theta d\varphi\,.\quad(18)$$

Total sound energy in the enclosure is determined by a sum of equations 6 and 18

$$E(t)=E_s(t)+E_d(t)\,.\quad(19)$$

In order to find diffuse energy at $t\to\infty$ we rewrite equation 9 for specular energy in the following way

$$E_s(t)\approx E_s^{(l)}(t)+E_s^{(d)}(t)+E_s^{(h)}(t)= =\frac{2}{\pi t^2}\frac{e^{-lt}}{dh}+\frac{2}{\pi t^2}\frac{e^{-dt}}{lh}+\frac{2}{\pi t^2}\frac{e^{-ht}}{ld}\quad(20)$$

and introduce values characterizing scattering properties of the walls

$$l_1=\frac{c}{2L}\left(\frac{\delta_3}{\gamma_3}+\frac{\delta_4}{\gamma_4}\right),\ d_1=\frac{c}{2D}\left(\frac{\delta_5}{\gamma_5}+\frac{\delta_6}{\gamma_6}\right),$$

$$h_1=\frac{c}{2H}\left(\frac{\delta_1}{\gamma_1}+\frac{\delta_2}{\gamma_2}\right).$$

From equation 18 one can find at $t\to\infty$

$$E_d(t)\approx\frac{l_1}{\frac{1}{T}-l}\left(E_s^{(l)}(t)-e^{-\frac{t}{T}}\right)+ +\frac{d_1}{\frac{1}{T}-d}\left(E_s^{(d)}(t)-e^{-\frac{t}{T}}\right)+ +\frac{h_1}{\frac{1}{T}-h}\left(E_s^{(h)}(t)-e^{-\frac{t}{T}}\right).\quad(21)$$

Note that equation 21 is not correct if value $1/T$ is close to l, d or h. We can see from equation 21 that diffuse energy strongly depends on specular energy. In order to diffuse energy prevails over specular energy it is required significant values of l_1, d_1, h_1 and small absorption of diffuse field $1/T<<l,d,h$.

In the enclosure with dimensions $H:D:L= =1:0.7:0.5$ considered above suppose that only one wall ($i=1$) can absorb sound and its absorption and reflection coefficients are equal $\alpha_1=0.7$, $\gamma_1=0.3$, $\delta_1=0$. Four walls scatter sound without absorption ($\alpha_3=\alpha_4=\alpha_5=\alpha_6=0$, $\delta_3=\delta_4=\delta_5=\delta_6=\delta$). The wall with the number $i=2$ does not absorb and scatter sound, i.e. $\gamma_2=1$. Figure 3 shows results of calculation of specular energy $E_s(t)$ defined by equation 6, diffuse energy $E_d(t)$ defined by equation 18 and total energy $E(t)$ defined by equation 19 for four values of the diffuse reflection coefficient $\delta=0,\ 0.05,\ 0.1,\ 0.2$. With increasing of scattering on the walls diffuse energy increases in relation to specular energy and the decay rate of total energy increases as well. If the diffuse reflection coefficient is great enough ($\delta=0.2$) diffuse energy dominates and the sound decay law tends to the Sabines's law. When scattering on the walls is small ($\delta=0.05$) total energy is defined only by specular energy and sound scattering is equivalent to sound absorption.

We see in Figure 3 specular energy and diffuse energy are approximately equal at $\delta=0.1$. Let us estimate value of the diffuse reflection coefficient which provides the similar decay law for both energies, i.e. $E_s(t)\sim E_d(t)$ at $t\to\infty$ in the considered enclosure with one absorbing wall and four scattering walls perpendicular to absorbing one. The smallest exponential decay rate of specular energy components given by equation 20 is $l=-c\ln\gamma_3\gamma_4/2L$. Because of $\gamma_3=\gamma_4=1-\delta$ the exponential decay rate of specular energy is equal $l\approx\delta c/L$ in case of small scattering $\delta<<1$. The exponential decay rate of diffuse energy is characterized by $T=4V/\overline{\alpha}cS$. In case of $H\sim L\sim D$ we can estimate $V\sim L^3$ and $S\sim 6L^2$. So $T\sim L/\overline{\alpha}c$

Specular and diffuse energies are approximately equal if their exponential decay rates are approximatly equal as well $l\sim 1/T$. Substituting here estimations for l and T we can find the following condition

$$\delta\sim\overline{\alpha}\,.\quad(23)$$

Equation (23) allows estimating the diffuse reflection coefficient essential for significant sound field diffusion in the enclosure. For example in a room with an absorbing ceiling $\alpha\approx 1$ diffuse energy is significant in comparison with specular energy if the diffuse reflection coefficient of walls is equal or greater then 0.1-0.2.

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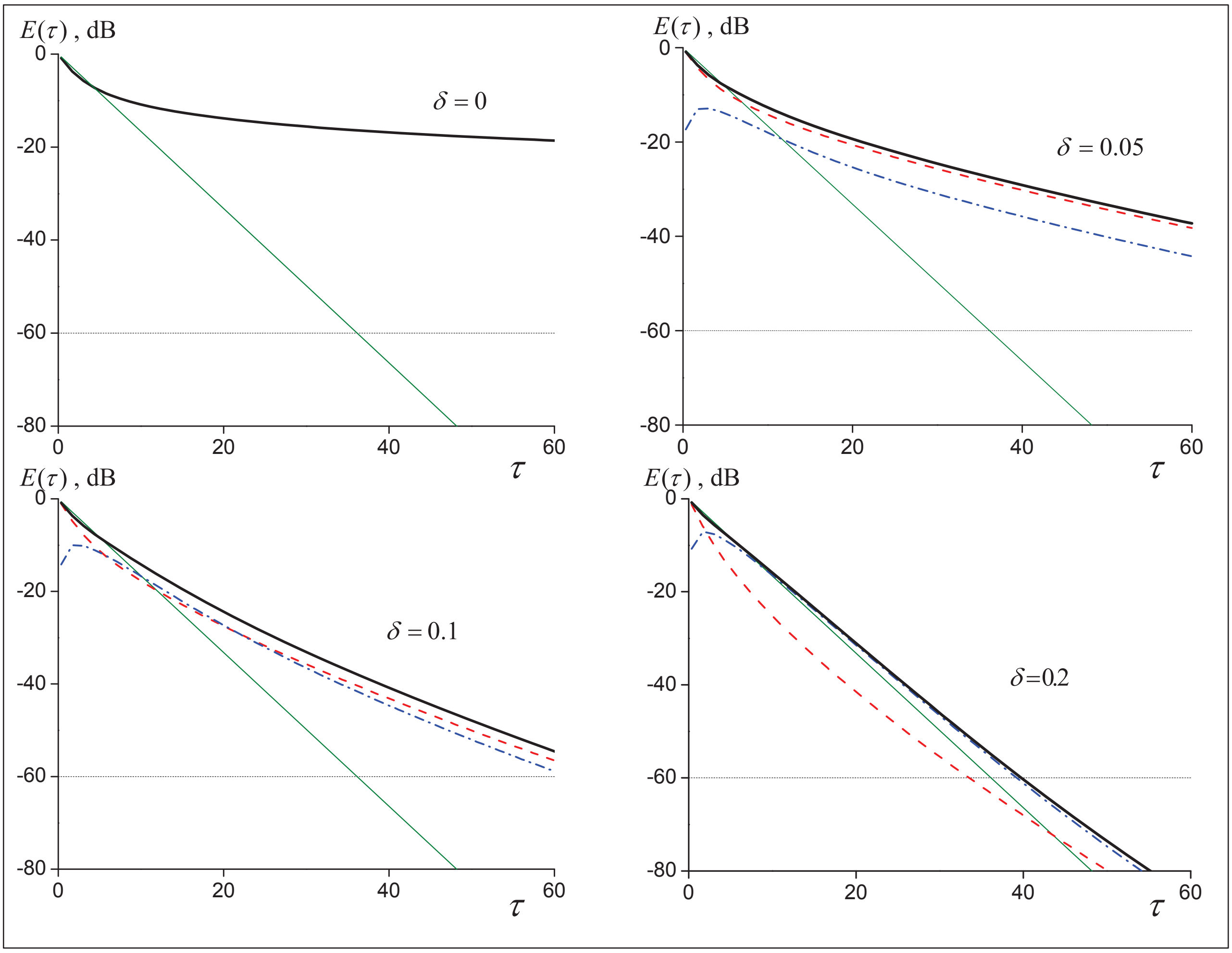

Figure 3. Decays of specular energy, diffuse energy and total energy in the rectangular enclosure with one absorbing wall and four scattering walls for different values of the diffuse reflection coefficient δ.
$E_s(t)$ (- - - - -), $E_d(t)$ (-·-·-·-), $E(t)$ (———), Sabine's law (———)

4. Conclusions

An analytical model of sound decay in a rectangular enclosure is proposed. In the model energy of speculary reflected sound and energy of sound scattered by enclosure walls are calculated separately. Decay of specular energy is nonexponential under any absorption distribution. Taking into account the scattering properties of walls permits to achieve an exponential decay law. It is shown that under nonuniform absorption distribution diffuse energy is significant if the diffuse reflection coefficient of the walls is close to the average absorption coefficient.

Application of the reverberation time is correct only for exponential decays. Whereas sound decay in a rectangular enclosure with specular reflecting walls is not exponential. So it seems that reverberation time formulas using only absorption coefficients are unjustified in calculation for rectangular enclosures and possibly for the enclosures with a poor randomizing strength.

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